1 Project Summary

First, describe and explain how ultrafilters can be used to construct ultraproducts, and the special case of ultrapowers (see [4]).

Next, briefly outline the structure of $\mathbb{R}$. Describe the ultrapowers of $\mathbb{R}$, $\mathbb{R}$. Comment on assumptions required for their existence, and their (non-)uniqueness [2], [1], [3].

Describe the notion of a theory, distinguishing first-order and higher-order theories. Briefly outline the first-order theory of $\mathbb{R}$. Describe the $*$ relation. Show that the first-order theory of $\mathbb{R}$ is that of $\mathbb{R}$.

Describe the Archimedean property. Briefly show that completeness (Every bounded set has a supremum) implies Archimedeanity, and so that $\mathbb{R}$ is Archimedean. Show that $\mathbb{R}$ is non-Archimedean. Hence show that completeness is not a first order property of $\mathbb{R}$.

2 Recommended Pre-Requisites

A course in logic; some basic knowledge of real analysis and point-set topology.

References


