Mathematics (BSc) 2nd Year Project:
Equivalents of Choice 2

N. J. H. Wontner

1 Project Summary

Define the Axiom of Choice (AC). Briefly comment on some of its uses in set theory and mathematics more widely.

Briefly explain how axioms can be equivalent (in the presence of other axioms). Comment on the notion of “independence”. Describe the following standard equivalents of AC: Zorn’s Lemma, Well-Ordering Principle, and Cardinal Comparison. Briefly outline the ways to show these are equivalent [4], [3], [2].

Define what it is for a set to be nowhere-dense, meagre, and comeagre. Prove the Baire Category Theorem (BCT) using AC for (a) complete metric spaces and (b) completely metrisable spaces. Comment on other spaces for which it holds.

Comment on how it is consistent with ¬AC that BCT fails. Define Dependant Choice (DC). Show that DC is equivalent to BCT [1].

Define the Baire Property for sets. Describe linear operators and Fréchet spaces. Define Baire measurability of linear operators. Show, using “all sets have the Baire property” and DC, that a Baire measurable linear operator from a Fréchet space to a normed space is continuous [5].

2 Recommended Pre-Requisites

A course in set theory, basic function analysis.

References