(1) (Hartogs’ Number) $|A| < |A| + h(A)$ for all $A$

(2) (Large Hierarchy) $|h(A)| < |\mathcal{P}(\mathcal{P}(A \times A))|$ for all $A$

(3) Prove directly that $\aleph_\alpha + \aleph_\alpha = \aleph_\alpha$ by expressing $\omega_\alpha$ as a disjoint union of two sets of cardinality $\aleph_\alpha$.

(4) (Ramsey Theory) Recall that $\aleph_\alpha \cdot \aleph_\alpha = \aleph_\alpha$ for every $\alpha$. Prove that:
   (a) $(\aleph_\alpha)^n = \aleph_\alpha$ for all $n \in \mathbb{N}$
   (b) $|[\aleph_\alpha]^n| = \aleph_\alpha$ where $[X]^n$ is the set of $n$-element subsets of $X$
   (c) $|[\aleph_\alpha]^{<\omega}| = \aleph_\alpha$ where $[X]^{<\omega}$ is the set of finite subsets of $X$

(5) (Cofinality) Prove that if $X = f[\omega_\alpha]$ for some function $f$, then $|X| \leq \aleph_\alpha$