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Talk Title: Adelic Geometry via Geometric Logic

Abstract:

On a syntactic level, the peculiarity of geometric logic can be seen from the choice of logical connectives used (in particular, we allow for infinitary disjunctions), but there are deep ramifications of this seemingly innocuous move. One, geometric logic is incomplete if we restrict ourselves to set-based models, but is complete if we also consider models in all toposes (i.e. not just Set) — as such, geometric logic can be viewed as an attempt to pull our mathematics away from a fixed set theory. Two, there is an intrinsic continuity to geometric logic, which is furnished by the definition of the classifying topos. Indeed, since every Grothendieck topos is a classifying topos of some geometric theory, this provides yet another way of viewing Grothendieck toposes as generalised spaces.

Both insights will inform the content of this talk. We shall start by giving a leisurely introduction to the theory of geometric logic and classifying toposes, before introducing a new research programme (joint with Steven Vickers) of developing a version of adelic geometry via topos theory.

The first step of this programme is to define the geometric theory of absolute values of Q and provide a point-free account of real exponentiation (which has already been completed). The next step is to construct the classifying topos of places of Q, which incidentally provides a topos-theoretic analogue of the Arakelov compactification of Spec(Z). This part is still work in progress, but some interesting observations (in particular, regarding whether we ought to view the Archimedean place as a point) have already emerged which we would like to share with the community. Fundamentally, we hope to use this framework to bring into focus certain essential aspects about how topology and algebra interact with one another, before extending our insights to explore various suggestive connections with the number theory.